On the use of stochastic simulations to explore the impact of human parameters on mass public shooting attacks

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ABSTRACT

A variety of individuals are likely to be involved in mass shooting attacks. However, the potential effect of people characteristics and response in such situations remains unclear. To address this issue, here we use a new stochastic model to identify patterns that maximize the survival probability and minimize the effectiveness of the shooter. As expected, while survival rates increase when people move fast (since they become a more difficult target to hit and their exposition time is reduced), chances of surviving decrease with non-escaping behaviours. We also found that densely occupied enclosures result in more casualties than sparsely occupied ones, however, casualties at high densities represent a smaller proportion of the individuals involved. Interestingly, even though the shooter effectiveness increases as the crowd becomes denser, so does too the survival probability overall. These findings challenge our current understanding of the impact of human parameters on mass shooting attacks.

Keywords: Mass shooting attacks; Stochastic simulation; Survival probability; Shooter effectiveness

1. INTRODUCTION

Although there is no universally accepted definition of mass shooting attack [1], in general, these events may be defined by the number of casualties produced, excluding the shooter(s). Common values are ≥ 3 [2], ≥ 4 [3] and ≥ 5 casualties [4]. A wide variety of definitions can affect estimates of mass shooting levels and trends. For instance, mass shootings derived from domestic and gang violence are contextually different from high-fatality indiscriminate terrorist killings in public venues [1]. In this study, we focus on higher-profile events motivated by mass murder in areas in which the perpetrator(s) can access a number of potential victims that are randomly selected [5]. Recent tragic examples include the 2015 Westgate shopping mall attack in Nairobi [6], the 2008 Mumbai attacks [7], the November 2015 Paris attacks [8], the 2017 Las Vegas Shooting [9], the 2016 Orlando nightclub shooting [10]. This kind of attacks have occurred and continue to occur around the world [5, 11, 12]. Since mass shootings might happen anywhere, at any time, it is imperative to increase people protection. Efforts from authorities and Law Enforcement Agencies include guidelines [4, 12, 13] and civilian training (e.g. "run, hide, fight" guidelines) [14-16] in addition to the security measures and precautions put in place (e.g. intelligence, access control, monitoring, deterrence methods, etc.). However, improving citizens protection is not an easy task as mass shooting attacks are based on a set of unpredictable factors that cause the results to vary according to chance. Due to the complexity of the subject matter, agent-based models have been proposed over the last few years. The idea behind these approaches is getting additional information from simulations to enhance understanding of a variety of aspects. For example, a model explored armed resistance in active shooting incidents at

educational institutions [17]. The study concluded that armed personnel is an efficient way to engage the active shooter (e.g. before the police arrive) thus reducing the potential number of casualties. Another study analysed unarmed resistance and found that even with a very low probability of overcoming a shooter, fighters may well save lives but put themselves at an increased risk [18]. Agent-based simulations were also used to examine gun control provisions (i.e. assault weapons and high-capacity magazines bans) [19]. The authors concluded that, the proposed law would have a small effect on the number of people shot. They found that the reduction of the rate of fire and the presence of security guards are likely to decrease the number of casualties. Law enforcement actions and civilian response strategies were analysed in another study focused on classroom environments [20]. This study showed that police response time has the largest impact on the number of casualties. But the more interesting finding was that there might not always be a clear optimal civilian response strategy (run, run-hide or hide). Another study examined the active shooter incidents casualty rates as a result of delays in event notification or police response [21]. The authors found that immediate evacuation of civilians, early detection of the shooter, and the rapid deployment of first responders are contributing factors in decreasing casualties. The most recent study explored the potential benefits of an integrated approach by combining human sensor data, building information and agent-based modelling [22]. Spatial coverage sensing data was varied from 0 to 100 % to assist simulated agents in effective evacuation plans. A positive correlation was found between the sensing coverage density and the total number of survivors.

Previous research used simulations to examine specific issues such as armed [17] and unarmed resistance [18], gun control [19], law enforcement response time [20, 21] and potentialities of real-time human sensor data [22]. In mass shooting attacks, people may face the attacker(s) in open areas without protection and/or decision support, at least during the first stages. However, the potential effect of population characteristics and people decisions and actions in such kind of scenarios remains unclear. The objective of this study thus was to investigate the impact of human parameters on the survival probability and shooter effectiveness in mass public shooting attacks. We argue that using stochastic simulations by a specialized model considering a small set of key human parameters is an appropriate starting point for (1) identifying patterns to improve self-protection and (2) exploring new approaches to simulate more complex processes and interactions. The proposed approach focuses on examining the outcome of an attack from the bottom-up rather than imposing prescribed situations. This contribution is expected to help advance the study of security and to open the way to new simulation and modelling tools based on two requirements: 1) providing enough detail in the model to allow sufficient accuracy and 2) fast simulation times (e.g. providing results faster than real-time). These tools should concentrate in the most essential parameters and interactions. Furthermore, due to the uncertainty related to the simulated scenario and the uncertainty related to the human behaviour, the use of Monte Carlo methods would allow the representation of all possible situations and the generation of samples of potential outcomes. We have claimed this new paradigm through several publications [23, 24].

2. METHOD

The proposed model comprises three elements: scenario, occupants and shooter. Spatial-temporal discretization is represented by a grid of scored cells (0.5 x 0.5 m) where agents move far from the attacker. Occupants have their own behavioural characteristics (e.g. response time, running speed) and decisions (e.g. stay or escape). The attacker uses an

AK-47 with several magazines (30 rounds) and fires random bursts (from 3 to 7 rounds) searching for the more densely occupied areas. Monte Carlo methods are used for the simulations. The model provides samples of the number of casualties and results of the survival probability and the shooter effectiveness.

2.1. Conceptual Model

The model considers the dynamic and random nature of the intended victims and shooter performances. Due to the complexity of the simulated event, several simplifications and assumptions were made. The idea was to create a simple model and exploring the implications before going on to more advanced inputs and functions. Hence, the selected options were defined to facilitate further improvements. Figure 1 shows the flow diagram of the model. The loop of iterations with the generation of the random variables and the initial conditions and the timing loop ensure the stochastic and the dynamic character of the proposed approach.
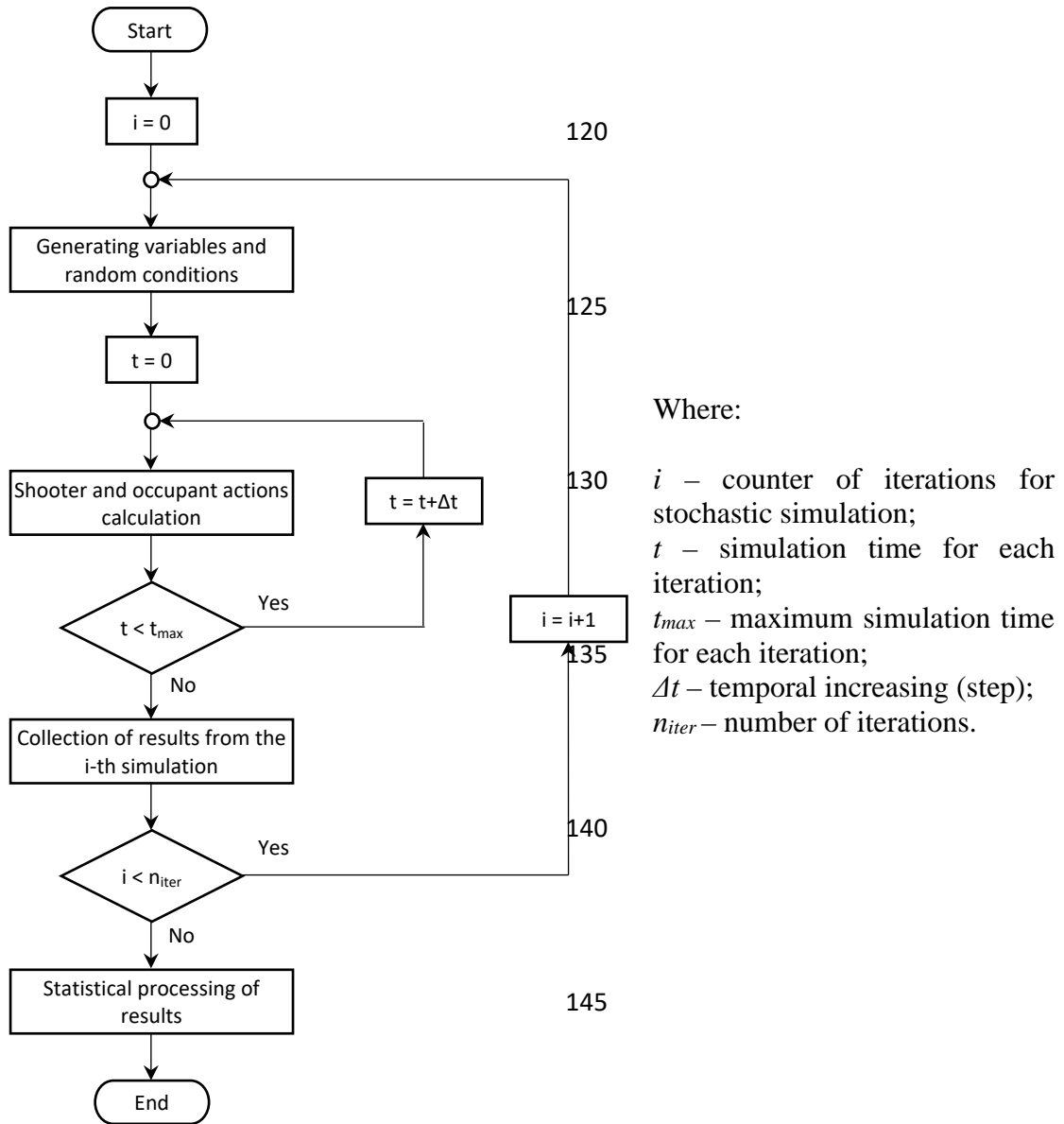

Where:

i – counter of iterations for stochastic simulation;
t – simulation time for each iteration;
t_{max} – maximum simulation time for each iteration;
$\varDelta t$ – temporal increasing (step);
n_{iter} – number of iterations.

Fig. 1. Flow diagram of the model.

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The model includes three main elements: the scenario, the occupants and the shooter.

- The scenario is defined as a rectangular enclosure (28 x 14 m) with three exits (see Figure 2). The space is divided into squared grid cells (0.5 x 0.5 m) that occupants move to and from. Each cell has a natural number that defines its potential. The lower the number the higher the potential (i.e. the number assigned to exit cells is 0 which is the higher potential). Occupants move towards cells according to Moore neighbourhood. Cells allow for only one occupant at a time. Shot occupants stop running and fall over occupying two cells that become unavailable for other escaping occupants. The shooter is assumed to be in the middle of the enclosure surrounded by cells with extremely low potential (high numbers) thus creating repulsive forces to the escaping occupants. In other words, numerical values are assigned to each squared cell in line with the travel distance to reach the exits (attraction) and to avoid the surrounding area of the shooter (repulsion). Three coordinate systems are used to represent the scenario (see Figure 2). The first one is a Cartesian coordinate system (x_1, y_1) with the origin $(0,0)$ in the bottom left-hand side. This system defines each grid cell to discretize the space in the floor plan. The second one is a Cartesian coordinate system (x_2, y_2) with the origin $(0,0)$ in the geometrical centre of the enclosure with uninterrupted counting axes in metres. The third one is a polar coordinate system (r, φ) with the origin in the geometrical centre of the enclosure. This system is used by the attacker when seeking and shooting targets.
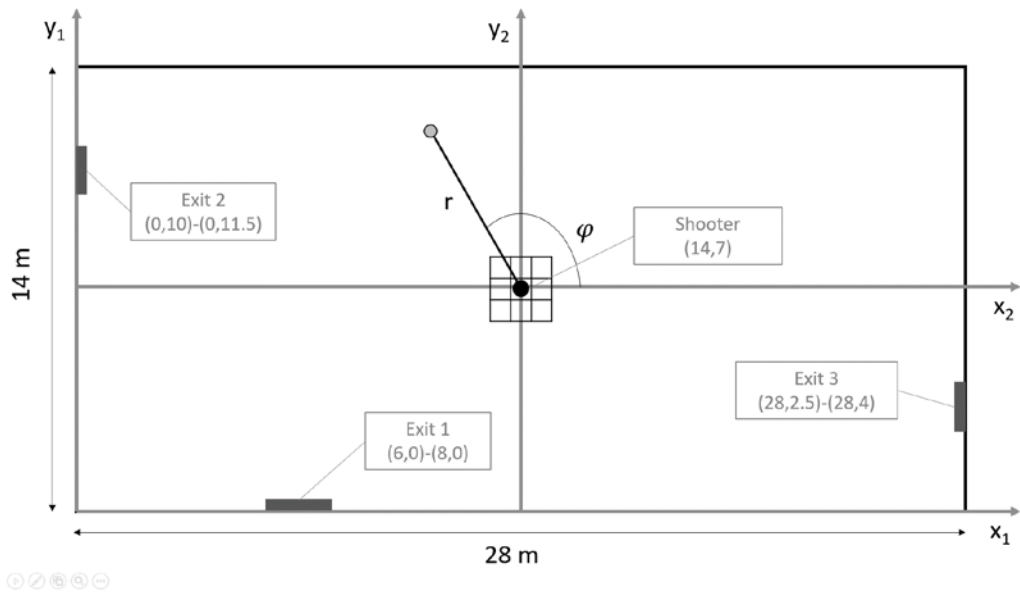

Fig. 2. Example of scenario.

- Occupants are randomly distributed in the enclosure. The behaviour of occupants is represented through two random variables. The first variable is the response time defined as the time from the first burst of the gunfire to the beginning of the purposive escape movement. In practice this variable is the sum of other two random variables: the duration of the first burst (a random value from 3 to 7 rounds at a given fire rate) and the reaction time of individuals assumed as a log-normally distributed variable with a mean of 5 s and standard deviation of 1 s. The second variable is the running speed, a user-adjustable parameter implemented according to a truncated normal distribution. The model has an additional parameter that can

be set to account for the proportion of individuals who do not escape during the attack (e.g. those who drop the ground, play dead, try to help others and/or simply stay paralysed). This is simulated by assigning a zero speed to the selected occupants. Each occupant has their own coordinates (polar and Cartesian) corresponding to the centre of the occupied cell and the minimum and maximum target angles representing the human torso (assumed 0.4 m in diameter) to determine whether they get shot or not. These angles are defined as follows:

$$\varphi_{max,min_j} = \varphi_j \pm \arctan(0.2/\rho_j) \tag{1}$$

Where φ_{max,min_j} is the maximum and the minimum target angles of *j-th* occupant, φ_j is the polar angle of the *j-th* and ρ_j is the radius vector of the *j-th* occupant. If the angle of the fired bullet is between the maximum and the minimum target angles, the occupant is struck with an estimated probability [18, 22]:

$$P_{hit} = \frac{100 - dist}{100} \times A_{Shoot} \tag{2}$$

Where *dist* is the distance between a shooter and the intended victim and A_{shoot} is the shot-accuracy assumed to 1.0 [25]. In the proposed scenario P_{hit} values ranged from 0.85 (at a maximum distance of 15 m) to 0.99 (at a minimum distance of 1 m). The average P_{hit} value of 0.92 was used for simulations.

- The shooter is assumed not to move in the centre of the enclosure. He uses an AK-47 with magazines of 30 rounds and fires in a standing shooting position short random bursts (between 3 and 7 rounds) with a combat rate of fire semi-auto of 40 rds/min and a muzzle velocity of 715 m/s. The time interval between bursts is the sum of the time spent by the shooter to choose the sector with higher number of occupants (assumed to be 5 s) and the time to turn and point the firearm with an angular rotation of 0.79 rad/s. If the magazine is empty a reload time of 4.5 s is added. The shooting angle is determined from the primary selection of a given sector. To do this, the scenario is divided into 12 hypothetical sectors of π/6 rad (30º), numbered between 0 and 11 counter-clockwise using the polar coordinates system (i.e. the gunman can turn 360 degrees). Then, the order of the sector with more individuals k_{max} is defined and the shooting angle φ_{shoot} is determined by:

$$\varphi_{shoot} = \pi \cdot k_{max}/6 + \pi/12 + U_1 + U_2 \tag{3}$$

Where $U_1 = U(-\pi/18, \pi/18)$ is the number generated with a uniform distribution used that represents the angle variation within the sector to the beginning of each burst and $U_2 = U(-\pi/60, \pi/60)$ is the number generated with a uniform distribution used for the angle of oscillation of each shoot. To sum up, the first aim of the shooter is the centre of the sector with more individuals and shoots with random lateral oscillations ranged between ±10º before each burst (alteration caused by the shooter) and between ±3º per round (alteration caused by the weapon recoil). This happens for every burst during the simulation.

The statistical processing of results is suggested here as part of the proposed method. It should be noted that this process covers results generated by several simulations for a given scenario (e.g. each scenario is run several times and the model generates a sample of casualties). The first output is the survival probability p_{sur} given by:

$$p_{sur} = \frac{n_{ocup} \cdot n_{iter} - n_{shot}}{n_{ocup} \cdot n_{iter}} \qquad (4)$$

Where n_{ocup} is the number of occupants in the scenario, n_{iter} is the number of iterations and n_{shot} the sum of occupants shot from all iterations.

The second output is the shooter effectiveness p_{effect} which is calculated as follows:

$$p_{effect} = \frac{n_{shot}}{n_{mag} \cdot n_{round} \cdot n_{iter}} \qquad (5)$$

Where n_{mag} represents the number of magazines used by the shooter and $n_{round} = 30$ is the number of rounds per magazine. The model provides the confidence intervals of both probabilities (p_{sur} and p_{effect}) by the Wilson method [26].

2.2. Computational Model

This computational model is an Object-oriented model (developed with Microsoft Visual C# 2017). Several classes were created: scenario, cells, occupants and input/output. The shooter was implemented as several functions of the class scenario.

The program has a window with user-adjustable parameters. The first parameter is the number of iterations allowing the number of stochastic simulations to be defined. Through the second parameter the user can define the number of occupants (i.e. people density at the time of the attack). The third parameter is the proportion of individuals who do not attempt to escape (e.g. dropping the ground, playing dead, trying to help others and/or simply staying paralysed). This may help to evaluate the consequences of non-escaping decisions. The fourth parameter is the number of standard 30 round magazines the shooter may use (4 by default). The fifth parameter is the assigned running speed to the occupants. The user can define the mean value (in m/s) of this random variable. Finally, the last parameter is the time available for the attack which may be used to investigate law enforcement response time. The potential impact of other parameters such as firing rate, number of rounds per magazine and individual reaction time can be configured by modifying the source code of the model.

The model allows 2-D visualization of the first iteration as part of the verification process (to check that the model does its job) to provide a better understanding of what is occurring. Figure 3 illustrates a visualization example where occupants are heading to the exits (spots in red) while some of them have been shot and are lying on the floor (spots in black, and purple) by the gunman located in the centre of the enclosure (blue square). The shooting direction is represented by a red line.
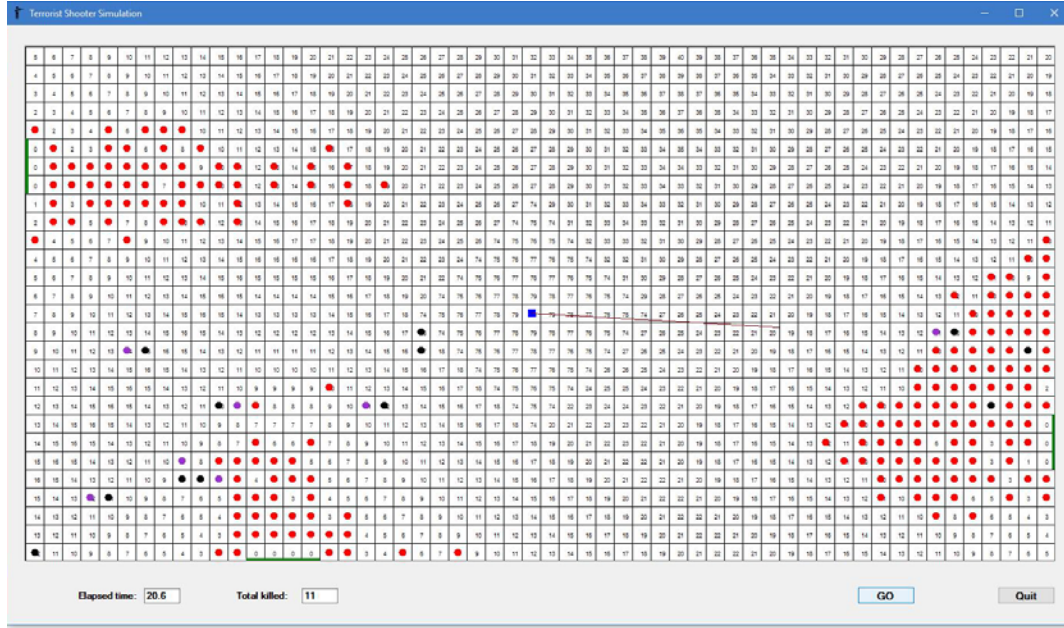
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Fig. 3. Visualization of the simulated scenario, the occupants and the shooter.

2.3. Simulation experiments

285 The relationship between inputs and outputs was analysed to test the coherence of the simulation approach and to examine key parameters related to the human scenario. One-at-a-time (OAT) sensitivity was used to analyse the influence of one parameter, keeping the others fixed and a regression analysis was conducted. The values of the input variables were varied as indicated in Table 1. The baseline case is represented with values in bold.

290 In total 250 iterations were run for each input combination (in total 10,750 runs). The time available for the attack (each iteration) was set to 360 s.

Table 1

Parameters considered for the simulation experiments. Values in bold indicate the baseline case. Note that
295 extreme input values may not be realistic but desirable for sensitivity analysis.

Parameter	Values	Notes
Mean value of running speed (m/s)	0.8; 1; 1.2; 1.4; 1.6; 1.8; **2**; 2.2; 2.4; 2.6; 2.8; 3; 3.2; 3.4; 3.6; 3.8; 4.	Mean value of a truncated normal distribution for running speed by considering the different population demographics .
Proportion of non-escaping occupants (%)	**0**; 5; 10; 15; 20; 25; 30;35; 40; 45; 50; 60.	Percentage of the population who do not escape (e.g. drop the ground, playing dead, or simply stay paralysed).
People density (per/m²)	0.8; 1; 1.2; 1.4; 1.6;1.8; **2**; 2.2; 2.4; 2.6; 2.8;2;2.5; 3; 3.5.	Directly related to the number of individuals involved in the attack (2 per/m² is equivalent to 784 occupants).

3. RESULTS

The first independent variable analysed was the running speed. This variable determines
300 the chance for individuals to reach a safe place before getting shot. Table 2 displays the statistics of casualties produced by the model. As expected, the faster the individuals move the more chance to flee. However, what is interesting from Table 2 is that the increase in running speed does not correspond with a constant rate in the number of

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casualties. Taking the baseline as a reference (running speed of 2 m/s with a mean 16 casualties), it is possible to see that a running speed of 1 m/s produced a mean of 24 casualties (an increase of 8 casualties) whereas a running speed of 3 m/s produced a mean of 13 casualties (a reduction of 3 casualties). Figure 4 shows the non-linear positive correlation between running speed and the survival probability. The P_{sur} values range from 0.965 to 0.985 denoting a small but a desirable improvement in the chances of surviving the attack. The slope of the survival probability is slightly higher between 0.8 and 2 m/s representing the potential vulnerability of scenarios involving individuals with reduced mobility showing that small increments on running speed would improve people protection. Equally, the increase in the running speed causes a decrease in the shooter effectiveness. The P_{effect} values in Figure 5 ranged from 0.23 to 0.10 representing a reduction in the shooter accuracy when individuals move faster. Also, the small increments to the running speed from 0.8 to 2 m/s have the greater impact on shooter effectiveness from 0.22 to 0.13. It is argued here that those who move fast turn themselves into more difficult targets. The perpetrator shoots towards density sectors rather than specific individuals. Transverse movement (more frequent in this scenario) and radial movement are possible in the model. Both movements may reduce the chances of being hit. But radial movement (i.e. running in a straight line away from the shoots) may be more dependent on the speed.

Table 2

Statistical characteristics of the number of casualties when changing the mean running speed. Baseline case in bold.

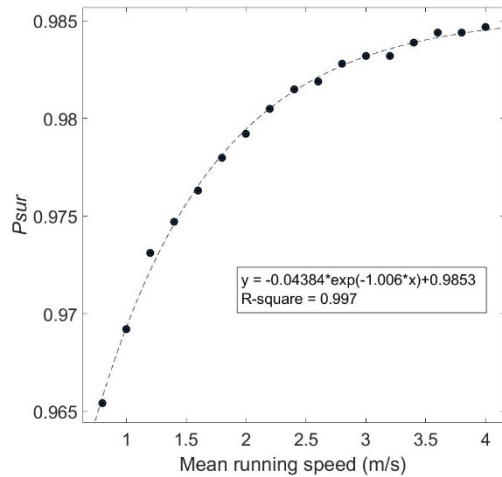| Mean of running speed (m/s)	Casualties				
	Mean	**S.D***	**Min.**	**Max.**	**95th percentile**
0.8	27	10.31	9	86	43
1	24	7.89	12	79	35
1.2	21	5.59	4	58	30
1.4	20	7.45	10	74	27
1.6	19	5.16	6	50	26
1.8	17	4.77	7	58	25
2	**16**	**3.50**	**8**	**29**	**22**
2.2	15	3.51	7	33	21
2.4	15	3.67	7	47	19
2.6	14	3.56	7	35	20
2.8	13	3.20	7	32	18
3	13	3.22	5	29	18
3.2	13	2.89	7	22	18
3.4	13	2.81	6	25	17
3.6	12	2.66	7	22	17
3.8	12	2.90	6	27	17
4	12	2.85	6	23	17

* S.D.- Standard Deviation

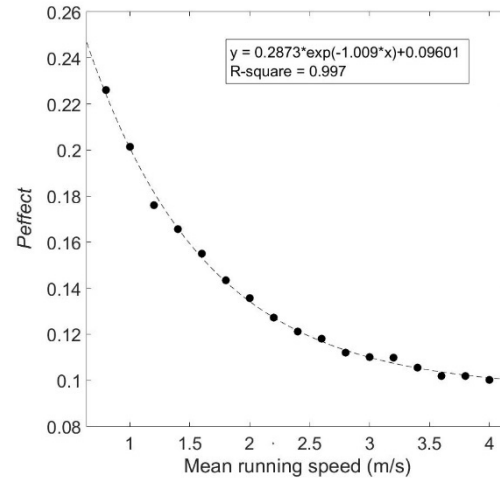**Fig. 4.** Running speed vs survival probability.

Fig. 5. Running speed vs shooter effectiveness.

The second independent variable analysed was the proportion of individuals who do not attempt to escape. In other words, this variable allowed us to know the potential impact of non-escaping behaviour. Table 3 illustrates the statistical characteristics of the number of casualties produced. Unsurprisingly, there was an increase in casualties when the assumed proportion of non-escaping individuals grew. On average 16 casualties with a range (8-29) were obtained from the baseline scenario (everybody escapes) while the worst-case scenario (60% of non-escaping individuals) resulted in 62 casualties with a range (40-97). Figures 6 and 7 show the scatter diagrams and linear correlations used to determine the relationship between the input and output variables. The rate in the proportion of non-escaping individuals corresponds with a constant change in the survival probability and the shooter effectiveness. Results reported in Figure 6 clearly indicate that non-escaping behaviour reduces the chances of surviving. Figure 7 shows that by varying the proportion of non-escaping individuals the likelihood of shooter effectiveness changes. The more individuals who do not attempt to escape the higher the P_{effect} values ranged from 0.13 to 0.51. However, it is important to note that the current version of the model does not reproduce other self-protective behaviours such as hiding or covering from fire.

Table 3
Statistical characteristics of the number of casualties when increasing the proportion of non-escaping individuals. Baseline case in bold.

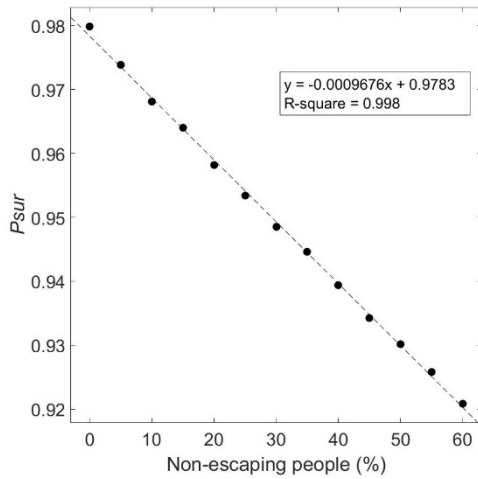| Proportion of non-escaping individuals (%)	Casualties				
	Mean	**S.D.**	**Min.**	**Max.**	**95th percentile**
0	**16**	**3.50**	**8**	**29**	**22**
5	21	4.41	6	49	27
10	25	5.09	15	54	33
15	28	4.55	17	44	36
20	33	6.72	18	76	42
25	37	6.57	22	77	47
30	40	6.28	25	67	51
35	43	6.77	27	82	54
40	47	7.92	32	86	61
45	51	8.93	34	95	66
50	55	9.28	37	101	69
55	58	8.39	41	96	73
60	62	9.42	40	97	81

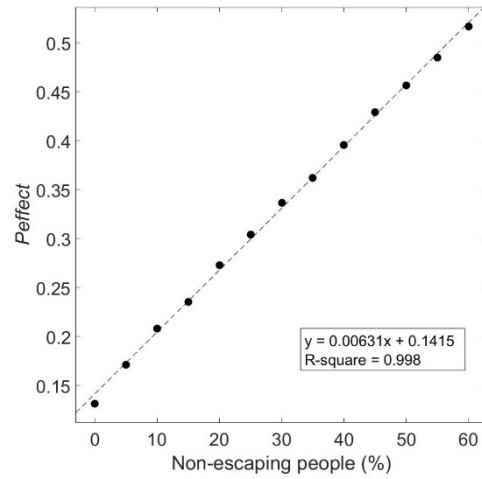**Fig. 6.** Proportion of non-escaping vs survival probability.

Fig. 7. Proportion of non-escaping vs shooter effectiveness.

The third variable refers to the number of potential victims at the time of the attack. The correlation between people density and model predictions is interesting because results can be interpreted into two ways. If we look at Table 4, it is easy to conclude that the more individuals involved in the attack the more casualties end up being. Yet another way to analyse results is by transforming the number of casualties into percentages. Surprisingly, from Table 5 it is possible to see that as the people density increases the percentage of casualties decreases. Figure 8 presents a non-linear positive correlation between the people density and the survival probability. The higher survival probability is produced when varying people densities < 1.75 per/m^2 (P_{sur} values from 0.93 to 0.97) whereas no significant increase was detected at higher densities (P_{sur} values from 0.97 to 0.98). Interestingly, as Figure 9 shows, we found a positive linear correlation between the number of intended victims (people density) and the shooter performance (shooter effectiveness).

Table 4
Statistical characteristics of the number of casualties when variying the people density (number of individuals involved). Baseline case in bold.

Population density (per/m^2)	Casualties				
	Mean	S.D.	Min.	Max.	95th percentile
0.25	7	2.00	1	11	10
0.5	9	2.41	3	19	14
0.75	11	2.52	5	21	15
1	12	3.84	3	48	17
1.25	13	3.45	5	35	18
1.5	14	3.13	7	28	19
1.75	15	3.25	7	31	19
2	**16**	**3.50**	**8**	**29**	**22**
2.25	17	3.96	7	36	23
2.5	18	3.78	9	31	25
2.75	20	4.20	8	42	27
3	21	6.37	8	91	28
3.25	22	4.77	10	56	29
3.5	22	4.72	12	48	29
3.75	24	5.85	14	81	31

Table 5
Statistical characteristics of the percentage of casualties when varying the people density (number of individuals involved). Baseline case in bold.

Population density (per/m^2)	Percentage of casualties				
	Mean	S.D.	Min.	Max.	95th percentile
0.25	6.70	2.04	1.02	11.22	10.20
0.5	4.80	1.23	1.53	9.69	6.91
0.75	3.70	0.86	1.70	7.14	5.10
1	3.09	0.98	0.77	12.24	4.34
1.25	2.60	0.70	1.02	7.14	3.67
1.5	2.30	0.53	1.19	4.76	3.23
1.75	2.12	0.47	1.02	4.52	2.77
2	**2.02**	**0.43**	**1.02**	**4.21**	**2.68**
2.25	1.93	0.45	0.79	4.08	2.56
2.5	1.86	0.39	0.92	3.16	2.55
2.75	1.82	0.39	0.74	3.90	2.50
3	1.75	0.54	0.68	7.74	2.38
3.25	1.70	0.37	0.78	4.40	2.24
3.5	1.61	0.34	0.87	3.50	2.11
3.75	1.66	0.40	0.95	5.51	2.11
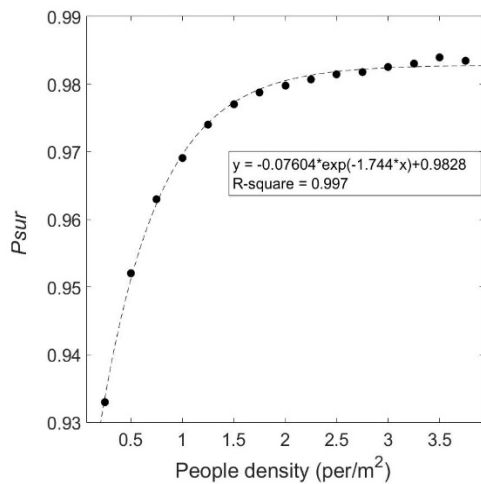
Fig. 8. People density vs survival probability.
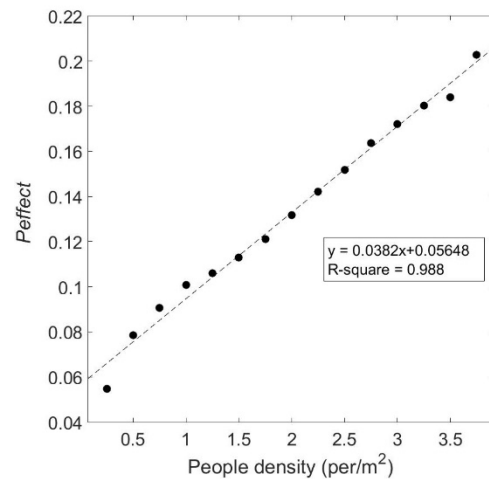
Fig. 9. People density vs shooter effectiveness.

4. DISCUSSION

The philosophy of creating a simple model and exploring the implications before going on to more sophisticated inputs and functions has been reflected in this paper. Previous studies in the field of mass shooting with ABM (Agent Based Models) have focused on pre-determined scenarios to examine the impact of imposed external conditions (from the top-down) on the outcomes (i.e. the potential casualties) [17-22]. Rather than starting with complex behaviours or imposing scenario conditions, it is simpler and more analytically robust to begin with key parameters (from the bottom-up). Using stochastic simulations, we analysed the impact of movement speed, non-escaping behaviour and people density on mass shooting attacks. Although rather tentative than definitive, together the results may provide insights into mass shooting attacks at crowded places.

To begin with, we found that the survival probability is higher when individuals move fast because they reduce the time in the 'danger zone' and become moving targets harder to hit. Similarly, the shooter effectiveness decreases when the running speed increases. Unquestionably, individuals are expected to move as fast as possible. Previous research assumed a mean running speed of 3.9 m/s while addressing the uncertainty assigning a wide dispersion of this variable with a standard deviation of 2.9 m/s and a minimum value of 0.30 m/s [18, 19]. In relation to movement speed, three factors can be considered when simulating mass shooting scenarios. The first factor is the movement behaviour. While some can sprint, others are likely to turn down, crawl and or crouch thus reducing their speed. The second factor is demographics. There can be individuals unable to escape or likely to be exposed longer (children, elderly, temporary mobility impairment, etc.). The third factor is the collective behaviour. The speed of movement is often dictated by that of the slowest member of the group (e.g. a parent with children). Other behaviours such as "helping others" may slow down movement and evacuation. However, these factors are not supported by empirical data yet. Video recording analysis of prior mass shootings and controlled experiments are highly desirable as this may prove an important issue for future research.

As expected, non-escaping behaviour reduces the chance of surviving and increases shooter accuracy. Previous research on a mass active shooter at a school building suggested that the all-run strategy might be the best when multiple exits are available [20]. Non-escaping individuals become targets easier to hit in scenarios without covers from fire, as observed in the shopping mall attack in Nairobi, Kenya 2013 [6]. There are internal and external factors that have an influence on non-escaping. Internal factors are related to the perception of the threat and/or emotional states often leading to the so-called `immobility reactions´ [28-30]. Being motionless may increase the chances to go unnoticed (e.g. when the attacker tends to shoot at moving targets), but this adaptive defensive behaviour involves longer exposure times. External factors are related to social and physical contexts. Social immediate context can prevent individuals from attempting to escape. Affiliation is the primary response to the perception of danger [27]. Rather than escaping, individuals seek familiar persons and places and try to help others. Another reason why people do not run/escape is that they cannot. Physical context such as congestion and jamming may prevent people from doing so. The proposed model represents the occurrence of jamming conditions at the exit doors because shot people block the way to others. In such situation, people are likely to `play dead´ as reported in the Orlando and Bataclan shooting attacks [8, 10]. Apart from the chance of surviving, one implication of non-escaping behaviour is that the situation may turn from a shooting attack to a hostage standoff where calm cooperation may result in being shot [18]. The simulation results confirm that, whenever possible, escape should be the top priority. The second priority should be to hide but only if escaping is not a safe or viable option (i.e. escaping can put individuals in a greater danger if it brings them closer to the shooter). Yet, hiding is not always a suitable option in open spaces such as vestibules or circulation areas. The last resort, mainly suggested in U.S.A., is to "fight". Previous research has shown that unarmed civilian resistance may save lives, but fighters put themselves at high risk [18] specially in open spaces. By contrast, in the UK and Europe, the option is to "tell" the police once individuals have reached a safe place. Clearly, the presented results support the idea that ensuring proper training to civilians could reduce the negative consequences of shooting attacks.

The number of potential victims is a random variable that depends on the scenario (e.g. shopping malls, religious buildings, mass gathering events, etc.) and the moment of the attack. Moreover, there is no apparent pattern or method for the selection of targets by attackers [31]. Each mass shooting presents differences in attacker characteristics, motivations and level of planning. Despite this, it is plausible to assume that "harm as many individuals as possible" is highly likely to be the major goal for the attackers that may well select crowded places. We found that sparsely occupied enclosures produce fewer casualties than densely occupied enclosures. But, contrary to expectations, the casualties at higher densities represent a smaller proportion of the total number of individuals involved. In other words, as people density increases the percentage of casualties decreases. Equally, the most striking result to emerge from the model is that the effectiveness of the shooter is higher but, paradoxically, the survival probability also increases as the crowd becomes denser. This is due to the combination of three factors. The first factor is the limited capacity of the shooter regarding firing rate and the number of magazines used. The second factor is the assumed shooter behaviour. Unlike other approaches (the shooter targets the closer occupants) [17-20], here the shooter seeks the most densely areas. In the simulated scenario, the crowd rapidly assembles to get through the exits forming three potential target areas. The more crowded the selected area the less likely the shooter turn and point the firearm to other areas where individuals can escape safely. The third factor is that bullets are not assumed to penetrate one person and hit another in the model, so the potential deadly capacity of rounds is lessened. Perhaps the impact of people density seems to be highly determined by the specific conditions of the scenario analysed. But this is undoubtedly an important parameter to consider when assessing mass shooting attacks.

This primary modelling effort has a number of limitations. The speed assigned to each occupant remains constant during the simulation. Therefore, no interruptions and/or variations in velocity are simulated. Due to space representation (cells of 0.5 x 0.5 m) the maximum people density is 4 per/m^2 and congestion is resolved by cell availability. In other words, an occupant will not move into a grid cell that is occupied by another occupant. Hence, the occupant will wait until the next cell is empty. Note that shot occupants are assumed to occupy two cells no longer available for escaping occupants. The model does not assign the flow through the exits based on the people density of space. The current model does not represent neither hard covers from fire (e.g. concrete walls or columns) nor soft covers for individuals to hide (e.g. furniture). The shooter has visual access to all intended victims. It is important to note that, as other previous works [18,19], the penetration of the bullets through individuals causing more than one casualty is not represented. The authors do not have the necessary anatomy knowledge to accurately predict or measure this. However, we agreed that these limitations should not impede the opportunity to analyse shooting attacks and generate valuable results [18, 19, 32, 33]. Importantly, no complex strategies or maneuverers (e.g. hiding, lying down, crawling, running in zig zag, etc.) to avoid getting shot are simulated. The escaping direction is determined by the minimum distances (i.e. individuals go towards the closer exits) and neither exit familiarity nor wayfinding behaviours are considered. Non-escaping individuals remain static. Whereas this behaviour is expected for some individuals, others may seek covers or places to hide. One of the main limitations in the model is that people are socially independent (e.g. they act regardless of the behaviour of others).

485   The model is not able to reproduce collective behaviours such as coordinated escape of familiar individuals and individuals helping others. What we know from the disaster literature is that affiliation is a primary response to the perception of danger [27]. While self-preservation is assumed to be the natural response to physical danger and perceived entrapment, expressions of mutual aid are common and often predominate. In real life,
490   people are not independent and there are social strains. Similarly, the model does not include fighting behaviour (neither armed nor unarmed resistance).

     The presented approach entails mathematical modelling and computer assisted investigation to understand the complexity of mass shooting attacks. Experts are invited
495   to criticize our assumptions and suggestions are welcomed. The main limitation is the lack of data from prior shootings and/or experimental research to appropriately validate the model. One alternative to cope with this setback would be interviews with experts in the area, or psychological research for a simple qualitative validation.

500   This study is the initial point for further research, including testing additional parameter combinations, new variables and scenarios. Future work will be oriented towards addressing the identified limitations. New scenarios potentially involve the introduction of covers from fire. Hard covers limit the shooter capacity and could be used as relative safe places for occupants and soft covers can reduce the struck probability. A further
505   important challenge would be to investigate and simulate other behaviours rather than simply "stay paralyzed or escape by the closer exit". For instance, route choice may be modified in accordance with different conditions such as congestion levels, shooter location, the presence of covers and exits familiarity. Hopefully, the next version of the model will allow modifying the movement targets (exits/covers) during the passage of
510   time. In future research, it may also be interesting to examine the impact of collective behaviour which have not been considered in this study. Setting up at a parameter that make certain agents invest in strategies that are employed by some other agents they are affiliated to is going to be crucial. The issue is how and why to make a social group move (or stay) in the model. Further improvements can also involve shooter capacity (e.g. more
515   than one shooter and different weapons, etc.) and behaviour. For instance, a further study with more focus on moving shooters (direction, speed, stops) is highly desirable.

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REFERENCES

[1]   Smart, R. Mass Shootings: Definitions and Trends. RAND Corporation. https://www.rand.org/research/gun-policy/analysis/supplementary/mass-shootings.html

[2]   Follman, M., Aronsen, G and Pan, D. (2017). U.S. Mass Shootings, 1982–2017: Data from Mother Jones' Investigation," Mother Jones, June 14, 2017.

http://www.motherjones.com/politics/2012/12/mass-shootings-mother-jones-full-data/

[3] Mass Shooting Tracker, homepage: https://www.massshootingtracker.org

[4] Chapman, S. et al. (2006). Australia's 1996 gun law reforms: faster falls in firearm deaths, firearm suicides, and a decade without mass shootings. Injury Prevention. 12 (6): 365–72. doi:10.1136/ip.2006.013714. PMC 2704353

[5] Australia-New Zealand Counter-Terrorism Committee. (2015) Active Shooter Guidelines for Places of Mass Gathering. ISBN: 978-1-922032-06-5.

[6] Anzalone, C. (2013). The Nairobi Attack and Al-Shabab`s Media Strategy. CTC Sentinel. Vol. 6 (10).

[7] Rabasa, A. et al. (2009). The Lessons of Mumbai. RAND Corporation. ISBN: 978-0-8330-4667-3.

[8] Brisard, J. (2015). The Paris Attacks and the Evolving Islamic State Threat to France. CTC Sentinel. Vol. 8 (11). 5-8.

[9] Lombardo, J. (2018). LVMPD Criminal Investigative Report of the 1 October Mass Casualty Shooting. LVMPD Event Number 171001-3519

[10] Straub, F. et al. (2017). Rescue, Response, and Resilience: A Critical Incident Review of the Orlando Public Safety Response to the Attack on the Pulse Nightclub. Critical Response Initiative. Washington, DC: Office Community Oriented Policing Services.

[11] Mullins, S. (2016). The Road to Orlando: Jihadist-Inspired Violence in the West, 2012-2016. CTC Sentinel Vol. 9 (6).26 -30.

[12] Smith, E. D. (2016). The profile of wounding in civilian public mass shooting fatalities. J Trauma Acute Care Surg Vol. 81(1).

[13] Interagency Security Committee (2015). Planning and Response to an Active Shooter: An Interagency Security Committee Policy and Best Practices Guide. (2015). https://www.dhs.gov/sites/default/files/publications/isc-planning-response-active-shooter-guide-non-fouo-nov-2015-508.pdf

[14] National Counter Terrorism Security Office. Counter Terrorism Policing. (2017). Crowded Places Guidance https://www.gov.uk/government/publications/crowded-places-guidance

[15] U.S. Department of Homeland Security Active shooter-Pocket Card Information. https://www.dhs.gov/publication/active-shooter-pocket-card

[16] NPCC: National Police Chiefs' Council. Run, Hide, Tell. https://www.npcc.police.uk/StaySafeAssets/Stay%20Safe_Transcript.pdf

[17] Anklam, C. et al. (2014). Mitigating Active Shooter Impact; Analysis for Policy Options Based on Agent/Computer Based Modeling. Journal of Emergency Management 13(3). DOI: 10.5055/jem.2015.0234.

[18] Briggs, T.W. & Kennedy, W.G. (2016). Active Shooter: An Agent-Based Model for Unarmed Resistance. Proceedings of the 2016 Winter Simulation Conference. IEEE Press. 3521-3531.

[19] Hayes, R., & Hayes, R. (2014). Agent-based simulation of mass shootings: Determining how to limit the scale of a tragedy. Jasss-the Journal of Artificial Societies and Social Simulation, 17(2), 1-12.

[20] Steward, A. (2017). Active shooter simulations: An agent-based model of civilian response strategy. Iowa State University. https://www.imse.iastate.edu/files/2014/03/StewartAlex-thesis.pdf.

[21] Lee, J. Y., Dietz, J. E. and Ostrowski, K. (2018). Agent-Based modelling for casualty rate assessment of large event active shooter incidents. Proceedings of

the 2018 Winter Simulation Conference M. Rabe, A. A. Juan, N. Mustafee, A. Skoogh, S. Jain, and B. Johansson, eds.

[22] Cho, C. Park, J. and Sakhakami. (2019). Emergency response: Effect of human detection resolution on risks during indoor mass shooting events, Safety Science 114, 160-170. https://doi.org/10.1016/j.ssci.2019.01.021

[23] Cuesta A., Alvear, D., Abreu O. and Silió D. (2014). Real-time Stochastic Evacuation Models for Decision Support in Actual Emergencies. Fire Safety Science 11: 1063-1076. 10.3801/IAFSS.FSS.11-1063

[24] Cuesta, A. Abreu, O. and Alvear, D. (2016). Future challenges in evacuation modelling, Evacuation modelling trends, Arturo Cuesta, Orlando Abreu and Daniel Alvear, Springer, UK. 103–131.

[25] Weaver, J.M. (1990). System Error Budgets, Target Distributions and Hitting Performance Estimates for General-Purpose Rifles and Sniper Rifles of 7.62 X 51 mm and Larger Calibers. USMAA Technical Report No. 461. U-S. Army material Systems Analysis Activity.

[26] Wilson, E. B. (1927). Probable inference, the law of succession, and statistical inference. Journal of the American Statistical Association. 22: 209–212.

[27] Mawson, A. R. (2005). Understanding Mass Panic and Other Collective Responses to Threat and Disaster. Psychiatry 68(2), 95-113

[28] Volchan, E., Rocha-Rego, V., Bastos, A.F., Oliveira, J.M., Franklin, C., Gleiser, S., Berger, W., Souza, G.G.L., Oliveira, L., David, I.A., Erthal, F.S., Pereira, M.G., Figueira (2017). Immobility reactions under threat: a contribution to human defensive cascade and PTSD. Neuroscience and Biobehavioral Reviews http://dx.doi.org/doi:10.1016/j.neubiorev.2017.01.025

[29] Kleine, R. A., Hagenaars, M. A. and van Minnen, A. (2018). Tonic immobility during re-experiencing the traumatic event in posttraumatic stress disorder. Psychiatry Research 270, 1105-1109. https://doi.org/10.1016/j.psychres.2018.06.051

[30] Santos M.F., Bastos A.F., Oliveira J.M., Figueira I, Gleiser S, Pereira M.G., Volchan E. and Erthal F.S. (2019) Hands Up! Atypical Defensive Reactions in Heavy Players of Violent Video Games When Exposed to Gun-Attack Pictures. Front. Psychol. 10:191. doi: 10.3389/fpsyg.2019.00191

[31] Osborne, J. R., and Capellan, J. A. (2015). Examining active shooter events through the rational choice perspective and crime script analysis. Security Journal 1-23. doi:10.1057/sj.2015.12

[32] Woodman, R. (2000). Agent-Based Simulation of Military Operations Other Than War Small Unit Combat. Unpublished Thesis: Naval Post Graduate School.

[33] Erlenbruch, T. (2002). Agent-Based simulation of German peacekeeping operations for units up to platoon level. Thesis. Naval Postgraduate School Monterey, California.

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